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Brane intersections in the presence of a worldvolume electric field

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Abstract: The study of brane intersections has provided important insights into a possible non-commutative structure of spacetime geometry. In this paper we focus on the D1⊥D3 system. We compare the D1 and D3 descriptions of the interesection and search for non-static solutions of the D3⊥D1 funnel equations in the presence of a worldvolume electric field. We find that the D1 and D3 descriptions do not agree. We find time dependent solutions that are a natural generalization of those found without the electric field.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Non-Commutative Geometry.

Contents

1. Introduction

The study of the low energy dynamics of D-branes in string theory, using the Dirac-Born-Infeld action, has produced many fascinating results [1−4]. In particular, the study of brane intersections has provided deep insights into a possible non-commutative structure of the geometry of space-time. These intersections exploit the possibility that a Dp-brane, coupling to the appropriate fluxes, can be interpreted as Dp'-branes of lower or higher dimensionality. In this context it is worth mentioning that study of systems where D-strings end on $D(2p+1)$ branes for $p = 1, 2, 3$ has given a concrete realization of non-commutative geometry, in the form of the so-called non-commutative fuzzy funnels [5−7]. Recent developments in this direction can be found in [8−12]. In particular, for the $D3 \perp D1$ system, the relevant geometry is that of the fuzzy two sphere.

One of the interesting aspects to emerge from studies of the $D(2p+1) \perp D1$ systems for $p = 1, 2, 3$, is that each system has two descriptions, which may be considered to be dual to each other. One description is the $D(2p+1)$-brane worldvolume theory which employs an abelian Dirac-Born-Infeld action with one excited scalar. The second description uses the D-string worldvolume theory. In the absence of a worldvolume electric field, these two descriptions are in complete agreement.

Recently time dependent solutions of the Dp-brane worldvolume theories have been considered in [8, 11]. In particular, in [8] space and time dependent fuzzy spheres in the $D(2p+1) \perp D1$ intersections of IIB string theory have been studied in detail. In the case of the $D3 \perp D1$ system one finds periodic space and time dependent solutions which can be described by Jacobi elliptic functions.

In this paper we consider the $D3 \perp D1$ system with an additional worldvolume electric field switched on. Our main interest is in the fate of the duality mentioned above, and in the fate of the time dependent solutions. We find that duality is completely lost; the space time dependent solutions have a nice generalization.

2. Duality in $D3\perp D1$-branes system in presence of electric field

In this section we study the intersection of a (p, q) string with a D3 brane. We describe this system as a D3⊥D1-brane system, with a worldvolume electric field switched on. For the D1⊥D3 system it is well known that the description using the D3 worldvolume theory is in complete agreement with the description employing the D1 worldvolume theory. Here, by "complete agreement" we mean that the two descriptions agree on both the profile of the funnel and the energy of the funnel. In this section we are interested in establishing whether a similar result holds even in the prescence of a worldvolume electric field. We will find that this is not the case. Further, by carefully considering where the two descriptions are valid we will be able to understand the disagreement.

Our starting point is the abelian Dirac-Born-Infeld action which describes the low energy dynamics of a single $D3$-brane

$$S_{\rm DBI} = \int L = -T_3 \int d^4\sigma \sqrt{-det(\eta_{ab} + \lambda^2 \partial_a\phi^i \partial_b\phi^i + \lambda F_{ab})}\,. \tag{2.1}$$

F_{ab} is the field strength of the $U(1)$ gauge field living on the brane worldvolume and $\lambda = 2\pi\alpha' = 2\pi\ell_s^2$ with ℓ_s the string length. We choose for simplicity to work in static gauge.

In the presence of a worldvolume electric field, the energy of the system is

$$\Xi = T_3 \int d^3\sigma \left[1 + \lambda^2\Big(\mid \nabla\phi \mid^2 + B^2 + E^2\Big) + \lambda^4\Big((B.\nabla\phi)^2 + (E.B)^2 + \mid E\wedge\nabla\phi\mid^2\Big)\right]^{1/2}. \tag{2.2}$$

We have traded the field strength for the electric field $F_{0i} = E_i$ and the magnetic field $B_r = \frac{1}{2}\epsilon_{rst}F^{st}$. It is straight forward to show that this energy can be brought into the following form

$$\begin{aligned}\Xi = T_3 \int d^3\sigma \Big[&\lambda^2 \mid \nabla\phi + \vec{B} + \vec{E}|^2 + \Big(1 - \lambda^2\nabla\phi.\,\vec{B}\Big)^2 - 2\lambda^2\,\vec{E}\,.\Big(\vec{B} + \nabla\phi\Big) + \\ &+ \lambda^4\left(\Big(\vec{E}\,.\,\vec{B}\Big)^2 + |\vec{E}\,\Lambda\nabla\phi\mid^2\right)\Big]^{1/2}.\end{aligned} \tag{2.3}$$

Requiring $\nabla\phi + \vec{B} + \vec{E} = 0$, Ξ reduces to $\Xi_0 \geq 0$ where

$$\Xi_0 = T_3 \int d^3\sigma \left[(1 - \lambda^2(\nabla\phi).\,\vec{B})^2 + 2\lambda^2\,\vec{E}\,.\,\vec{E}) + \lambda^4((\vec{E}\,.\,\vec{B})^2 + |\vec{E}\Lambda\nabla\phi\mid^2)\right]^{1/2}. \tag{2.4}$$

For a typical solution of the system

$$\phi = \frac{N_m + N_e}{2r}\,; \qquad \vec{B} = \frac{N_m\hat{r}}{2r^2}\,; \qquad \vec{E} = \frac{N_e\hat{r}}{2r^2} \tag{2.5}$$

where N_m and N_e are integers. We can evaluate Ξ_0 to obtain

$$\Xi_0 = T_3 \int d^3\sigma \left[\left(1 + \lambda^2\frac{N_m(N_m + N_e)}{4r^4}\right)^2 + 2\lambda^2\frac{N_e^2}{4r^4} + \lambda^4\frac{N_m^2 N_e^2}{16r^8}\right]^{1/2}$$

which can be further reduced to the expression

$$\Xi_0 = T_3 \int d^3\sigma \left[1 + \lambda^4 \frac{N_m^2[(N_m+N_e)^2 + N_e^2]}{16r^8} + 2\lambda^2 \frac{N_m(N_m+N_e)}{4r^4} + 2\lambda^2 \frac{N_e^2}{4r^4}\right]^{1/2} . \quad (2.6)$$

This provides the description of the system using the D3 brane worldvolume theory.

We now develop a description which employs the D1 brane worldvolume theory. By turning on a worldvolume electric field, we can use the D-string theory to provide a description of the (p,q) string we are considering. For N_m D-strings ending on a $D3$-brane with a background $U(1)$ electric field the action becomes [5]

$$S = -T_1 \int d^2\sigma STr \left[-det(\eta_{ab} + \lambda^2 \partial_a \phi^i Q^{-1}_{ij} \partial_b \phi^j + \lambda F_{ab}) det Q^{ij}\right]^{1/2} . \quad (2.7)$$

For the case that we consider, the field strength $F_{ab} = EI_{ab}$ with I_{ab} the $N_m \times N_m$ identity matrix. Notice that the D-string description involves a non-abelian theory which is to be contrasted with the abelian D3-brane description. The ansatz for the Higgs fields is

$$\phi_i = \hat{R}(\sigma)\alpha_i \, . \quad (2.8)$$

where α_i's are generators of an $N_m \times N_m$ representation of the SU(2) algebra

$$[\alpha_i, \alpha_j] = 2i\epsilon_{ijk}\alpha_k \, .$$

Evaluating the action for this ansatz, we obtain

$$S = -T_1 \int d^2\sigma STr \left[(1 - \lambda^2 E^2 + \alpha_i\alpha_i \hat{R}'^2)(1 + 4\lambda^2 \alpha_j\alpha_j \hat{R}^4)\right]^{1/2} . \quad (2.9)$$

Minimizing this action with respect to R for fixed field strength E, we obtain the following solution

$$\phi_i = \frac{\alpha_i}{2\sigma\sqrt{1-\lambda^2 E^2}} \, . \quad (2.10)$$

A comment is in order: one should properly make the ansatz at the level of the equations of motion and not at the level of the action and this solution is indeed consistent with the equations of motion.

This solution is similar to the dual description of the D3$\perp$D1-system: the scalar field ϕ^i describes a fuzzy two sphere of radius

$$R(\sigma) = \lambda \sqrt{\frac{Tr[\phi^i(\sigma)]^2}{N_m}} = \frac{N_m \pi \ell_s^2}{\sigma\sqrt{1-\lambda^2E^2}} \sqrt{1 - \frac{1}{N_m^2}} \quad (2.11)$$

$$\approx \frac{N_m \pi \ell_s^2}{\sigma\sqrt{1-\lambda^2E^2}} \, . \quad (2.12)$$

The D3 brane is located at $\sigma = 0$ where the radius of the fuzzy sphere diverges. Introduce electric displacement D conjugate to E, which is, by definition

$$D = \frac{1}{N_m}\frac{\delta S}{\delta E} = \frac{\lambda E}{g\sqrt{1-\lambda^2E^2}} = \frac{N_e}{N_m} \, . \quad (2.13)$$

It is clear that $\frac{\lambda E}{\sqrt{1-\lambda^2 E^2}} = g\frac{N_e}{N_m}$, so $N_e \to \infty$ when $\lambda E \to 1$. Consequently, for sufficiently large but fixed N_m, if $N_e \to \infty$ for fixed $R(\sigma)$ we find $\sigma \to \infty$. This suggests that we should identify $\sigma \leftrightarrow \lambda\phi$ and $R \leftrightarrow r$ and which fixes

$$N_e + N_m = \frac{N_m}{\sqrt{1-\lambda^2 E^2}}$$

for $N_e + N_m = N \to \infty$. Thus for large N, $\sqrt{1-\lambda^2 E^2} \to$ some fraction $\frac{m}{n}$ or equivalently $\lambda E \to \sqrt{1-(\frac{m}{n})^2}$ where $0 < \frac{m}{n} < 1$. Further from the definition of D we have

$$N_m\sqrt{(\frac{n}{m})^2 - 1} = gN_e\,.$$

Thus $N_e \to \infty$ whenever $m \to 0$.

Having matched the profiles of the two descriptions, we now ask if the energies of the two descriptions match. The energy of the (N_m, N_e) system is given by

$$\Xi = T_1\int d\sigma\sqrt{N_m^2 + g^2N_e^2} + T_3\left(1 - \frac{1}{N_m^2}\right)^{-1/2}\int dR 4\pi R^2\,. \qquad (2.14)$$

We might expect that this energy matches the energy of D3 brane description in the large N_m limit where both descriptions may be valid. This is indeed the case for the situation with no electric field switched on.

In the presence of an electric field, in the large N_m limit for fixed N_e, the energy from the D-string description, equation (2.14), becomes $T_1 N_m \int d\sigma$. The energy computed using the dual D3 description uses (2.6). We only need to consider the contribution from the second term of the integrand which is $T_3\int d^3\sigma\lambda^2\frac{N_m\sqrt{(N_m+N_e)^2+N_e^2}}{4r^4}$ and using $\phi = \frac{N_m+N_e}{2r}$ this can be recast as $T_1\frac{N_m\sqrt{(N_m+N_e)^2+N_e^2}}{N_m+N_e}\int d\sigma$. Again for fixed N_e, but in the large N_m limit, $\frac{\sqrt{(N_m+N_e)^2+N_e^2}}{N_m+N_e}$ becomes 1, so we have agreement from the two sides. But we can also take the large N_m limit keeping N_e/N_m fixed at any arbitrary $K > 0$, then (2.14) becomes $T_1 N_m\sqrt{1+gK^2}\int d\sigma$ while for (2.6) again we can only consider the contribution from the second term and we find it as $T_1\frac{N_m\sqrt{(1+K)^2+K^2}}{1+K}\int d\sigma$. Thus in general we have a disagreement, i.e the presence of electric field spoils the duality between the D3 and D1-brane descriptions of the D3$\perp$D1 system.

To understand why the two descriptions do not match, we will now consider the regimes of validity of the two descriptions. Both descriptions will receive higher derivative corrections. Our strategy is to estimate the range of $R(\sigma)$ for which these corrections can safely be ignored. Indeed, the corresponding analysis in the absence of an electric field shows that the two descriptions have a large overlap in their domain of validity. In the presence of an electric field, we find that this is no longer the case.

The higher derivative corrections to the Born-Infeld action arise as terms in the ℓ_s expansion of the low energy effective theory. Due to the presence of the electric field, we find that the leading corrections are two derivative terms which take the form $\ell_s^5 F_{ab}\partial^a\partial^b\phi$. In the absence of the electric field the leading corrections is the four derivative term $\ell_s^6(\partial^2\phi)^2$. We now require

$$\ell_s^5 F_{ab}\partial^2\phi \ll \ell_s^4(\partial\phi)^2\,.$$

In the D-string description this inequality becomes $R(\sigma) \gg N_m \ell_s^2 E$. For the $D3$ brane description, the inequality is $R(\sigma) \gg \frac{N_m+N_e}{\ell_s E}$. The overlap between the two descriptions in the limit $N_m \to \infty$ limit (for fixed electric field) does not grow. This is in contrast to the case with no electric field where the overlap in the limit becomes large. This strikingly different behaviour can be traced back to the term $\ell_s^5 F_{ab}\partial^a\partial^b\phi$ whose existence is due to the prescence of the background electric field.

3. Non-static $D3\perp D1$ funnels in presence of electric field

Time dependent solutions of the $D3\perp D1$ funnel equations have been studied in detail in [8]. In this section we wish to study the time dependent $D3\perp D1$ funnel solutions in the presence of a worldvolume elecric field. Consider N D-strings ending on a $D3$-brane in the presence of an elecric field. This system has the action (see (2.7))

$$S = -T_1 \int d^2\sigma STr\left[-\det(\eta_{ab} + \lambda^2\partial_a\phi^i Q_{ij}^{-1}\partial_b\phi^j + \lambda F_{ab})\det Q^{ij}\right]^{1/2} \tag{3.1}$$

in which the field F_{ab} is antisymmetric in the a, b indices and $F_{\tau\sigma} = EI_N$ where I_N is the $N\times N$- identity matrix. Inserting the space-time dependent ansatz

$$\phi_i = \hat{R}(\sigma,\tau)\alpha_i \tag{3.2}$$

into the above action, where the α_i's are again the generators of an $N\times N$ representation of the SU(2) algebra

$$[\alpha_i, \alpha_j] = 2i\epsilon_{ijk}\alpha_k\,,$$

we obtain

$$S = -T_1 \int d^2\sigma Str\sqrt{1+\lambda^2 c\hat{R}'^2 - \lambda^2 c\dot{\hat{R}}^2 - \lambda^2E^2}\sqrt{1+4\lambda^2 c\hat{R}^4}\,. \tag{3.3}$$

We have made use of the result $\sum_{i=1}^3(\alpha^i)^2 = c = N^2-1$. To express the above action in terms of dimensionless variables we consider the following rescalings

$$r = \sqrt{2\lambda\sqrt{c}}\hat{R} \qquad \tau = \sqrt{\frac{2}{\lambda\sqrt{c}}}\tau \qquad \sigma = \sqrt{\frac{2}{\lambda\sqrt{c}}}\sigma \qquad \lambda E = e\,.$$

The rescaled action is

$$\tilde{S} = -\int d^2\sigma\sqrt{1+r'^2-\dot{r}^2-e^2}\sqrt{1+r^4}\,. \tag{3.4}$$

The conserved (rescaled) energy is

$$T_{\tau\tau} = \Xi = (1+r'^2-e^2)\sqrt{\frac{1+r^4}{1+r'^2-\dot{r}^2-e^2}}\,, \tag{3.5}$$

and the conserved pressure is

$$T_{\sigma\sigma} = (1-\dot{r}^2-e^2)\sqrt{\frac{1+r^4}{1+r'^2-\dot{r}^2-e^2}}\,, \tag{3.6}$$

with dots and primes implying differentiation with respect to the rescaled time and space respectively and to simplify the notation we rename $\tilde{\tau}$, $\tilde{\sigma}$ as τ, σ.

We wish to study the purely time dependent solutions of the equation (3.5) and hence we drop the derivatives of r with respect to σ. We take the initial condition $r(0) = r_0$ and $\dot{r}(0) = v$. From the rescaled energy expression we get

$$\dot{r}^2 = \frac{r_0^4(1-e^2) + v^2 - r^4(1-v^2-e^2)}{1+r_0^4}. \tag{3.7}$$

In terms of

$$r_1^4 = r_0^4 + \frac{v^2(1+r_0^4)}{1-v^2-e^2} \tag{3.8}$$

we can integrate to obtain

$$\int_0^\tau d\tau' = \sqrt{\frac{1+r_0^4}{1-v^2-e^2}} \int_{r_0}^{r} \frac{dr'}{\sqrt{r_1^4 - r'^4}}. \tag{3.9}$$

This gives

$$i\tau\sqrt{\frac{1-v^2-e^2}{1+r_0^4}} = \int_{r_0}^{r_1} \frac{dr'}{\sqrt{r'^4 - r_1^4}} + \int_{r_1}^{r} \frac{dr'}{\sqrt{r'^4 - r_1^4}}. \tag{3.10}$$

Now, using the notation

$$T = cn^{-1}\left(\frac{r_0}{r_1}, \frac{1}{\sqrt{2}}\right) \tag{3.11}$$

the solution can be written as

$$r(\tau) = r_1 cn\left(\bar{\tau} + T, \frac{1}{\sqrt{2}}\right), \tag{3.12}$$

where $\bar{\tau} = i\tau\sqrt{2}\sqrt{\frac{1-v^2-\lambda^2 e^2}{1+r_1^4}}$. The interpretation of r_1 is now clear: it is the amplitude of the solution, and it controls the brane radius at the instant in time for which the collapsing speed is zero. Further, (3.12) implies that we can always cast the time dependence of the brane radius in terms of an amplitude and T, a cn function, which can be interpreted as the initial phase of the solution. Using (3.8), the energy of this solution can be written as

$$\Xi = \sqrt{1+r_1^4}.$$

Consider the cases when $v = 0$ and $e = 0$ separately. For $v = 0$ we find (21) becomes

$$\dot{r}^2 = \frac{r_0^4(1-e^2) - r^4(1-e^2)}{1+r_0^4}, \tag{3.13}$$

while $r_0 = r_1$ and T vanishes.

If $e = 0$ (3.7) becomes

$$\dot{r}^2 = \frac{r_0^4 + v^2 - r^4(1-v^2)}{1+r_0^4}. \tag{3.14}$$

We find that now r_1 and r_0 differ when $T = cn^{-1}(r_0/r_1, 1/\sqrt{2})$. The case where both $e = 0$ and $v = 0$ has been discussed in the reference [8].

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Consider the equations (3.7), (3.13) and (3.14). We want to examine the solutions in the limit $r_0 \to \infty$, or equivalently, $r_1 \to \infty$. In this limit we find $\dot{r}^2 = 1 - e^2$ from (3.7) and (3.13); in contrast, from (3.14) and for the case where both $e = 0$ and $v = 0$ we have $\dot{r}^2 = 1$. Thus, in the presence of a worldvolume electric field, the brane collapses at a speed which is less than the speed of light; this speed has been reduced by a factor of $\sqrt{1-e^2}$, and further, the collapsing speed does not depend on the initial speed.

In the same way, it is also interesting to study purely spatial dependent solutions $r(\sigma)$; in this case we drop all τ derivatives. Starting from the expression for the conserved pressure, consider $r(\sigma = 0) = r_0$ and $r'(\sigma = 0) = u$. From (3.6) we have

$$r'^2 = \frac{r^4(1+u^2-e^2) - r_0^4(1-e^2) + u^2}{1+r_0^4}, \tag{3.15}$$

which can be integrated to obtain

$$\int_0^\sigma d\sigma' = \sqrt{\frac{1+r_0^4}{1-u^2-e^2}} \int_{r_0}^{r} \frac{dr'}{\sqrt{r'^4 - r_1^4}}, \tag{3.16}$$

where

$$r_1^4 = r_0^4 - \frac{u^2(1+r_0^4)}{1+u^2-e^2}.$$

This can be expressed as

$$\begin{aligned}\frac{\sigma\sqrt{2}r_1\sqrt{1+u^2-e^2}}{\sqrt{1+r_0^4}} &= cn^{-1}\left(\frac{r_1}{r}, \frac{1}{\sqrt{2}}\right) - cn^{-1}\left(\frac{r_1}{r_0}, \frac{1}{\sqrt{2}}\right), \\ &= cn^{-1}\left(\frac{\frac{r_1^2}{rr_0} + \frac{1}{2}\sqrt{(1-(\frac{r_1}{r})^4)(1-(\frac{r_1}{r_0})^4)}}{1 - \frac{1}{2}(1-(\frac{r_1}{r})^2)(1-(\frac{r_1}{r_0})^2)}, \frac{1}{\sqrt{2}}\right).\end{aligned} \tag{3.17}$$

If $u = 0$ we find (3.15) becomes

$$r'^2 = \frac{r^4(1-e^2) - r_0^4(1-e^2)}{1+r_0^4}, \tag{3.18}$$

$r_1 = r_0$ and (3.17) becomes

$$\frac{\sigma\sqrt{2}r_0\sqrt{1-e^2}}{\sqrt{1+r_0^4}} = cn^{-1}\left(\frac{r_0}{r}, \frac{1}{\sqrt{2}}\right). \tag{3.19}$$

Further for $e = 0$ (3.15) is modified to

$$r'^2 = \frac{r^4(1+u^2) - r_0^4 + u^2}{1+r_0^4} \tag{3.20}$$

and

$$r_1^4 = r_0^4 - \frac{u^2(1+r_0^4)}{1+u^2}. \tag{3.21}$$

The case of $e = 0$ and $u = 0$ has been studied in [8].

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Note that for $r_0 = u = 0$ or for $u^2 = r_0^4(1-e^2)$, from (3.15) we find

$$r'^2 = r^4(1-e^2)\,. \tag{3.22}$$

This implies

$$\mp r = \frac{1}{\sigma\sqrt{(1-e^2)} \mp \frac{1}{r_0}}\,. \tag{3.23}$$

Thus for particular initial conditions of r' and r, we do not find the infinite periodic brane-anti-brane array, but rather a solution with a decaying r. In the presence of an electric field it is attenuated by the factor of $1/\sqrt{1-e^2}$. The equation $r'^2 = r^4(1-e^2)$ provides a generalization of the BPS equation in presence of an electric field. Note that for $e = 0$ we get the usual BPS equation $r'^2 = r^4$. The BPS equation can be obtained by studying the condition to have an unbroken supersymmetry, which amounts to the condition for the vanishing of the variation of gaugino on the world volume of the intersection [8]. A second way to obtain it, is to ask for minimality of (3.5) for $e = \dot{r} = 0$, as

$$T_{\tau\tau} = \Xi = \sqrt{(1+r^4)(1+r'^2)} = \sqrt{(r^2 \pm r')^2 + (1 \mp r^2 r')^2}$$

and minimality gives $r'^2 = r^4$.

For equations (3.7) and (3.15), consider the Wick rotation $\tau \to i\sigma$. This provides interesting results in the cases where e, v and u are all zero. Here we find (3.7)$\to$ (3.15) for $\tau \to i\sigma$ and $v \to iu$. Thus in the presence of the electric field, the space time dependent solutions generalize nicely.

Another interesting feature of these solutions is the $r \leftrightarrow 1/r$ duality, which arises as the consequence of the invariance of the complex curve under an $r \leftrightarrow 1/r$ automorphism. More precisely, in the absence of any worldvolume electric field and with $v = 0$, we find (3.7) reduces to the equation of a complex curve $s^2 = \frac{r_0^4 - r^4}{1+r_0^4}$ where we have defined $\dot{r} = s$ and r and s are interpreted as complex variables. For this curve the relevant automorphisms have been studied in [8] and a connection to the $r \leftrightarrow 1/r$ duality has been made. Here we find a similar duality. Towards this end, we study an automorphism of the curve (3.7) for $e \neq 0$ and $v \neq 0$. After defining $s = \dot{r}$ we find

$$s^2 = \frac{r_0^4(1-e^2) + v^2 - r^4(1-v^2-e^2)}{1+r_0^4}\,. \tag{3.24}$$

The automorphism acts as $r \to R = \frac{1}{r}$, $r_0 \to R_0 = \frac{1}{r_0}$, $s \to \tilde{s} = \frac{is}{r^2}$, $v \to v' = -ivR_0^2$. This automrphism acts at fixed e. For $v = 0$, $r \to R = \frac{r_0^2}{r}$, $s \to \tilde{s} = \frac{isr_0^2}{r^2}$, is still a good automorphism of (3.24).

4. Conclusion

In presence of an electric field the duality between the D1 and D3 description is no longer valid. We have further argued that the D-string description breaks down. This is perhaps not surprising. Indeed, in this limit [13] have argued that the effective tension of the string goes to zero. Thus, excited strings modes will not be very heavy compared to massless string modes and one might question the validity of the Dirac-Born-Infeld action which retains only the massless modes.

Further in the presence of a world volume electric field, space time dependent solutions can be generalised nicely and for $r_0 = 0$ we observe brane collapses with a speed less than that of light. We have also obtained a generalization of the BPS solution. An automorphism of our solutions relevant for the $r \leftrightarrow 1/r$ duality has been discussed.

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